PBL: Rotation & Oscillation
活動單元: 轉動與振盪

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Names	ID (last 7 digits)	Gr

0. Introduction 簡介

In this activity unit, you will perform virtual experiments with four different public domain physics simulations dealing with PBL: Rotation & Oscillation. Before you start answering the questions, play with each simulation. Get familiar with each of the different effects, buttons and tabs of the animations.

在這項活動中，您將使用涉及轉動與振盪的四個不同的公共領域物理模擬執行虛擬實驗。在你開始作答問題前，玩玩看每個模擬實驗，熟悉動畫的每個不同效果，按鈕和標籤。

1. Rotational Motion Review 轉動運動回顧

1.1 Download, Run and Play with the PhET Simulation: “Ladybug Revolution”. 下載，運行和玩 PhET 模擬: 「旋轉的瓢蟲」

1.2 [Select: Intro] Allow the ladybug to spin. What aspects of the variables ($\bar{\mathbf{s}}$, $\bar{\mathbf{v}}$, $\bar{\mathbf{a}}$) are constant? What aspects are changing? [選擇: 入門] 讓瓢蟲旋轉。 變量 (s, v, a) 的哪幾項是常數? 哪些會發生變化(s, v, a)?

Constant 常數:	Change 變化:

1.3 [Select: Intro]What is the direction of the acceleration? What is the direction of the velocity? [選擇: 入門] 加速度的方向是什麼？ 速度的方向是什麼？

$\bar{\mathbf{a}}$:	$\bar{\mathbf{v}}$:

1.4 Systematic Experimentation 系統實驗

a. [Select: Rotation] Keep ω Constant. Find the relationship between the $|\bar{\mathbf{a}}(t)|$ and the distance (r) from the center of the turntable. (HINT: Use the ruler to measure lengths) [選擇: 旋轉] 將角速度 (ω) 設為定值。找出 $|\bar{\mathbf{a}}(t)|$ 和旋轉半徑 (r) 的關係。(提示: 使用尺量測長度)

Distance (r) 距離	**$\lvert \bar{a}(t) \rvert$**	**Graph 圖形**	**Fit to equation 擬合方程式**
			(ω=_____)

b. [Select: Rotation] Keep ω Constant. Find the relationship between the $|\bar{\mathbf{v}}(t)|$ and the distance (r) from the center of the turntable. (HINT: Use the ruler to measure lengths) [選擇: 旋轉] 將角速度 (ω) 設為定值。找出| $\bar{\mathbf{v}}$(t)) |和旋轉半徑 (r) 的關係。(提示: 使用尺量測長度)

Distance (r) 距離	**$\lvert \bar{v}(t) \rvert$**	**Graph 圖形**	**Fit to Equation 擬合方程式**
			(ω=_____)

c. [Select: Rotation] Keep r Constant. Find the relationship between the $|\bar{\mathbf{a}}(t)|$ and the angular velocity (ω) of the turntable. [選擇: 旋轉] 旋轉半徑 (r) 設為定值。找出| $\bar{\mathbf{a}}$(t)) |和角速度 (ω) 的關係。

ω	$\|\bar{a}(t)\|$	Graph 圖形	Fit to Equation 擬合方程式
			(r=_____)

d. [Select: Rotation] Keep r Constant. Find the relationship between the $|\bar{\mathbf{v}}(t)|$ and the angular velocity (ω) of the turntable. [選擇: 旋轉] 旋轉半徑 (r) 設為定值。找出$|\bar{\mathbf{v}}(t))|$和角速度 (ω) 的關係。

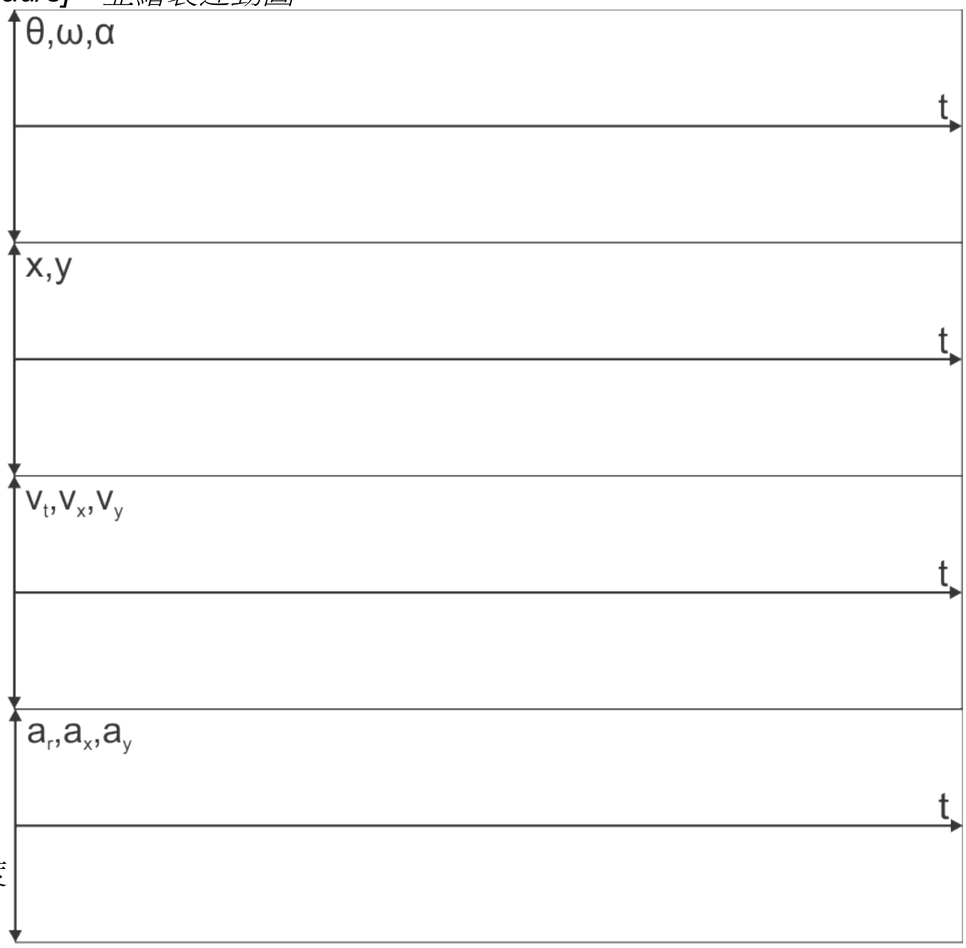
ω	$\|\bar{v}(t)\|$	Graph 圖形	Fit to Equation 擬合方程式
			(r=_____)

e. Combine the above relationships you found in questions a,b,c,d to give mathematical equations relating acceleration (radial) and velocity (tangential) to angular velocity (ω) and the distance from the center of the turntable (r). 結合以上的關係式 (a,b,c,d)，給出關於加速度 (徑向)、速度 (切線)、角速度 (ω) 和圓半徑 (r) 的數學方程式。

$a_t(\omega,r) = 0$ [m/s^2]	$v_t(\omega,r) =$
$a_r(\omega,r) =$	$v_r(\omega,r) =$

1.5 [Select: Rotation] Set the angular velocity to 10 [rad/s]. Draw graphs of the motion. *[選擇: 旋轉] 將角速度設為 10 [rad/s]，並繪製運動圖。*

a. Draw 繪製 ($\theta(t),\omega(t),\alpha(t)$)
b. Draw Position 繪製位置 ($s_x(t),s_y(t)$)
c. Draw tangential (v_t) and the x,y components of velocity $\bar{\mathbf{v}}$. 繪製切線 (v_t) 和速度 $\bar{\mathbf{v}}$ 的 x,y 分量
d. Draw radial (a_r) and the x,y components of acceleration $\bar{\mathbf{a}}$. 繪製徑向 (a_r) 和加速度 $\bar{\mathbf{a}}$ 的 x,y 分量
e. Write equations (motion as f(t) to describe the graphs you have drawn as a function of θ=ωt.) (Remember: ω is constant) 寫出方程式 (運動為 f(t) 來描述你所繪製的圖形是 θ=ωt 的函數)。(記住: 角速度 (ω) 是常數)



<table>
<tr><td>$\theta(t) =$</td><td>$s_x =$

$s_y =$</td></tr>
<tr><td>$\omega(t) =$</td><td>$|\bar{\mathbf{v}}| =$

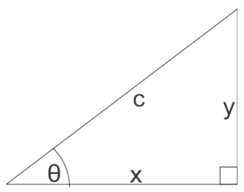
$v_x =$

$v_y =$</td></tr>
<tr><td>$\alpha(t) =$</td><td>$|\bar{\mathbf{a}}| =$

$a_x =$

$a_y =$</td></tr>
</table>

f. For the right-angle triangle (below), what is the value of x, y and c in terms of θ. 依照下圖的直角三角形，以 θ 表示，寫出 x, y 和 c。

<table>
<tr><td rowspan="3">
</td><td>x =</td></tr>
<tr><td>y=</td></tr>
<tr><td>c =</td></tr>
</table>

2. Torque 力矩

2.1 Download, Run and Play with the PhET Simulation: “Torque” 下載，運行和玩 PhET 模擬: 「力矩」

2.2 [Select: Intro] Apply a force to start the turntable spinning. What happens after the force is removed? [選擇: 入門] 施力使圓盤轉動，當你不再施力時會發生什麼呢?

2.3 [Select: Torque] Apply different forces ($\bar{\mathbf{F}}$) at different offsets ($\bar{\mathbf{r}}$) from the pivot. At 2.5s, set $\bar{\mathbf{F}}\rightarrow 0$. Continue to run the simulation. [選擇: 力矩] 在與樞軸不同的偏移量 ($\bar{\mathbf{r}}$) 處施加不同的力量 ($\bar{\mathbf{F}}$) 。在 2.5s 時，設置 F→0。繼續運行模擬。

a. Record in the following table the values (for the bug) after the force is set to 0 (i.e. t> 2.5s). 將力設為 0 後, 在下表中記錄數值 (對於蟲子) (例如: t> 2.5s)

F (N)	r (m)	τ (Nm)	v_t (m/s)	|a| (m/s/s)	a_t (m/s/s)	a_r (m/s/s)	θ(2.5s)	ω	α
1	1								
1	2								
2	1								

b. Which is the key parameter determining the final rotational speed? 決定最終轉速的關鍵參數為何?

c. How does the direction of $\bar{\mathbf{a}}$ differ between (a) when the force is applied (b) when there is no torque? 加速度 ($\bar{\mathbf{a}}$) 的方向在 (a)當施加力時 (b)無力矩 有何不同?

(a)	(b)

d. How does $\bar{\mathbf{a}}$ change during the time the force is applied? 在施力期間，加速度 ($\bar{\mathbf{a}}$) 有什麼變化嗎?

2.4 Systematic Experimentation: Accelerating Bug [Select: Torque] 蟲子加速時的系統實驗 [選擇: 力矩]

a. Apply a force ($\bar{\mathbf{F}}$) at different offsets ($\bar{\mathbf{r}}$) from the pivot. Record in the following table the values at t=2.5s. 在距樞軸不同的偏移量 ($\bar{\mathbf{r}}$) 處施加恆定的力 ($\bar{\mathbf{F}}$)。在下表中記錄 t = 2.5s 時的數值。

F (N)	r (m)	τ (Nm)	v_t(m/s)	|a| (m/s/s)	a_t (m/s/s)	a_r (m/s/s)	θ(2.5s)	ω(2.5s)	α
1									
1									
1									

b. Apply a different forces ($\bar{\mathbf{F}}$) at constant offset ($\bar{\mathbf{r}}$) from the pivot for 2.5 seconds. Record in the following table the values at t=2.5s. 從樞軸以恆定的偏移量($\bar{\mathbf{r}}$) 施加不同的力 ($\bar{\mathbf{F}}$) ，持續 2.5 秒。在下表中記錄 t = 2.5s 時的數值。

F (N)	r (m)	τ (Nm)	v_t(m/s)	|a| (m/s/s)	a_t (m/s/s)	a_r (m/s/s)	θ(2.5s)	ω(2.5s)	α
	1								
	1								
	1								

c. [Select: Torque, Force in any direction] Keeping |$\bar{\mathbf{F}}$| and |$\bar{\mathbf{r}}$| constant, change the direction φ of applied force. [選擇: 力矩, 在任意方向的力] 保持 |$\bar{\mathbf{F}}$| 和 |$\bar{\mathbf{r}}$| 不變，改變施加力的方向 (φ)。

F(N)	r(m)	angle(φ)	τ(N-m)	a_t (m/s/s)	ω(2.5s)	α
1	1					
1	1					
1	1					

d. Suggest an equation: (i) τ = f(|r|,|F|,φ). Suggest an equation for α = f(τ). 建議一個方程式: (i) τ = f(|r|,|F|,φ)。 為 α = f(τ)，建議一個方程

2.5 Theory and Drawing Conclusions 理論與得出結論

a. How the angular acceleration (α) or torque (**τ**) be maximized? 如何最大化角加速度 (α) 或力矩 (**τ**) ?

1	2	3

b. Consider the results of the experiments you have done, try to write equations relating the angular acceleration (α) to Force (F), offset from the pivot (r) and/or torque (τ). 考慮以上你完成的實驗結果，嘗試編寫將角加速度 (α) 與力 (F) ，樞軸的偏移量 (r) 和/或力矩 (τ) 有關的方程式。

c. The constant in this equation has the name: "Moment of Inertia" which we will denote with the symbol I. We will explore this constant more in the next sections of the PBL. From the equation you have discovered relating angular acceleration to the forces and torques, what must be the units of "Moment of Inertia"? 上述方程式中的常數名為: “轉動慣量"，我們將用符號 "I" 來表示它，我們也將在 PBL 的下一部分中進一步探討此常數。從方程式中您發現了角加速度與力和力矩的關係，因此“轉動慣量”的單位必須是?

2.6 [Select: Moment of Inertia] Change each of the parameters, one by one, to find out their effect on angular acceleration (α) and thus moment of inertia (**I**) (Suggestion: Keep Torque (τ) Constant, e.g. τ=1) [選擇: 轉動慣量] 分別更改每個參數，以了解它們對角加速度 (α) 以及轉動慣量的影響 (**I**) (建議: 保持力矩(τ)恆定，例如: τ= 1）

mass (kg)	**r_{inner} (m)**	**R_{outer} (m)**	**α (rad/s)**	**I**
0.12	0	4		

a. Effect of changing the Platform Mass 改變平台質量的影響 ______________
b. Effect of changing the Inner Radius 改變內部半徑的影響 ________________
c. Effect of changing the Outer Radius 改變外半徑的影響 _________________

2.7 [Select: Angular Momentum] [選擇: 角動量]

a. Keep m, R_{inner}, R_{outer} constant. Find the relationship between the | $\overline{\mathbf{L}}$ | and the angular velocity ($\overline{\boldsymbol{\omega}}$) 將 m, R_{inner}, R_{outer} **設為定值，找出角動量 (| $\overline{\mathbf{L}}$ |)和角速度($\overline{\boldsymbol{\omega}}$)的關係**

$\overline{\boldsymbol{\omega}}$	$\overline{\mathbf{L}}$	**Graph 圖形**	**Equation 方程式**

b. Keep ω constant. Find the relationship between the | $\overline{\mathbf{L}}$ | and the moment of inertia (I) 將 ω **設為定值，找出角動量(| $\overline{\mathbf{L}}$ |)和轉動慣量 (I) 的關係**

I	$\overline{\mathbf{L}}$	**Graph 圖形**	**Equation 方程式**

c. Combine your two relationships into a single equation to provide $\overline{\mathbf{L}}(\omega)$ 將上述實驗結果綜合成 $\overline{\mathbf{L}}(\omega)$ 的方程式

$\overline{\mathbf{L}}(\overline{\boldsymbol{\omega}})$=

3. Oscillations: The Pendulum 振盪: 鐘擺

3.1 Download, Run and Play with the PhET Simulation: “Pendulum Lab” 下載，運行和玩 PhET 模擬: 「鐘擺實驗室」

3.2 [Select: Intro, gravity] Start the pendulum moving and then set the gravity to g=0. How can we describe this motion? What similarities does it have with the previous simulations? [選擇: 介紹,重力] 開始擺盪運動，將重力 (g) 調整為 0。我們如何形容這項運動呢？ 描述此運動與之前模擬的運動有何相似之處?

3.3 [Select: Intro] Set the gravity to that of earth Earth. Draw a graph of pendulum's y-axis movement vs time (horizontal axis). [選擇: 介紹] 將重力同整為地球的重力。畫出鐘擺的 y 軸（縱向）運動與時間（橫軸）的曲線圖

y

t

3.4 [Select: Intro] What does this plot show? Any similarities with mathematical formulas for periodic motion that we have seen before? [選擇: 介紹] 該圖顯示了什麼？與我們之前看到的周期性運動的數學公式有甚麼相似之處嗎？

3.5 Systematic Experimentation [Select: Intro] 系統實驗: [選擇: 介紹]

a. Find the relationship between the | $\overline{\omega}$ | and the mass of the pendulum (m). **找出角速度 (| $\overline{\omega}$ |) 和擺錘重量 (m) 的關係**

m (kg)	**$\overline{\omega}$ (rad/s)**	**Graph 圖形**	**Equation 方程式**

b. Find the relationship between the | $\overline{\omega}$ | and the Length of the pendulum (L). **找出角速度 (| $\overline{\omega}$ |) 和擺錘長度 (L) 的關係**

l (m)	**$\overline{\omega}$ (rad/s)**	**Graph 圖形**	**Equation 方程式**

c. Find the relationship between the | $\overline{\omega}$ | and the force of gravity ($\overline{F}_g$). **找出角速度 (| $\overline{\omega}$ |) 和重力 ($\overline{F}_g$) 的關係 (Hint: 藉由改變星球改變重力)**

$\overline{F}_g$ (N)	**ω (rad/s)**	**Graph 圖形**	**Equation 方程式**

d. Try to combine your mathematical expressions to get a single equation describing the angular frequency of the pendulum. 嘗試結合你的數學表達式來獲得一個描述擺角頻率 ($\overline{\omega}$) 的方程式。

$\overline{\omega}$ =

3.6 Theory and Drawing Conclusions 理論與得出結論

a. While the pendulum is in motion, change the mass of the pendulum. (i) What happens? (ii) Why? 當鐘擺運動時，改變鐘擺的質量。(i) 發身什麼了? (ii) 為甚麼？

(i)	(ii)

b. While the pendulum is in motion, change the force of gravity. (i) What happens? (ii)Why? 當鐘擺運動時，改變重力。(i) 發身什麼了? (ii) 為甚麼？

(i)	(ii)

4. Masses and Spring 質量與彈簧

4.1 Download, Run and Play with the PhET Simulation: “Masses and Springs” 下載，運行和玩 PhET 模擬: 「質量與彈簧」

4.2 Systematic Experimentation 系統實驗

a. Select: [Energy] [Damping large] Find the value of the spring constant when the slider is on small spring constant and large spring constant. Remember that $|\mathbf{F}_{spring}| = k\ \Delta x$ and $\Sigma\bar{\mathbf{F}}=0$ at equilibrium. 選擇: [能量][阻尼最大] 找出當滑塊在小彈簧常數和大彈簧常數時的彈簧常數。記得: Fspring = k Δx 以及平衡時 ΣF=0

1. Mass 質量 = _____

2. Δx	Equation 方程式	k
		Small:
		Large:

b. Select: [Energy] [Damping large] Find the relationship between spring constant k and angular frequency ω. (Hint: $\omega=2\pi f$, $f=1/T$). 選擇: [能量][阻尼最大] 找出彈簧常數 (k) 和角頻率 (ω) 之間的關係 (提示: $\omega=2\pi f$, $f=1/T$)

1. Choose Mass 選擇質量 (m) = _____

2. k	ω	Graph 圖形	Equation 方程式

c. Now we know the oscillation equation is $x=c\cos(\theta)$. If we want to rewrite the equation with angular frequency ω. What will you describe? 現在我們了解震盪公式為 $x=c\cos(\theta)$，如果我們要用角速度重寫公式，該如何表示?

$s_x=$ $c\cos(\theta)$ $s_y=$	$s_x=$ $s_y=$
$v_x=$ $v_y=$	$v_x=$ $v_y=$
$a_x=$ $a_y=$	$a_x=$ $a_y=$

5. 您的意見 Student Comments

5.1 Did you enjoy the activity? Choose one 你喜歡這個活動嗎？選一個

▫ HATED 憎恨 ☐ 25% ▫ 50% 馬馬虎虎 ☐ 75% ▫ LOVED 喜爱

Why? 為什麼？

5.2 Suggest one or two additional questions that could be asked concerning any of the simulations. (If we add your question, you will get 1% bonus marks for the course!) 提出 1 或 2 個關於任何模擬的其他問題。（如果你的問題被使用，你將獲得該課程 1%的獎勵分數！（最多加 5 分））

Activity 活動	**Suggested Question 建議的問題**	**Answer to suggested question 回答建議的問題**

5.3 有沒有別的意見？ Any other suggestions to improve this activity?
